Generalised Co-ordinates

Let there be a particle or system of n - particles moving under possible constraints, then there will be a minimum no. of independent co-ordinates are required to specify the motion of particle or system of particles. The set of independent co-ordinates sufficient in number to specify the system configuration is called generalised co-ordinates.

Generalised co-ordinates are denoted by $q_1, q_2 \ldots q_n$.

The number of generalised co-ordinates is equal to the number of degrees of freedom of the system. If a system has n degrees of freedom, then the number of generalised co-ordinates is also equal to n.

Degrees of freedom

The no. of independent variables/co-ordinate necessary to specify the position of a dynamical system is known as degrees of freedom.

In case of a system of n- particles, there are $3n$ ~~3N~~ independent variables/cartesian co-ordinates. If there exist holonomic constraints expressed by l equation, then the equation will eliminate l co-ordinates out of $3n$.

Thus $(3n-l)$ are generalised co-ordinates which is also known as degrees of freedom.

D.O.F $\boxed{f = 3n - l}$

(i) A free particle, DOF = 3

(ii) N free " DOF = $3N$

these are $q_1, q_2 \ldots q_n$ each of which may depend upon time. If the system contains N particles, then the Cartesian co-ordinates of these N particles (x_1, y_1, z_1), $(x_2, y_2, z_2) \ldots (x_N, y_N, z_N)$ can be expressed in terms of $q_1, q_2 \ldots q_n$ which may also depend on time.

The transformation equations are

$$x_1 = x_1(q_1, q_2 \ldots q_n, t), \quad y_1 = y_1(q_1, q_2, \ldots q_n, t)$$

$$\ldots\ldots$$

$$z_n = z_n(q_1, q_2 \ldots q_n, t)$$

these can be combined into a single equation,

$$\boxed{\vec{r_i} = \vec{r_i}(q_1, q_2 \ldots q_n, t)}$$ where $i = 1, 2, 3 \ldots N$

Generalised displacement :-

We have, $\vec{r_i} = \vec{r_i}(q_1, q_2 \ldots q_n, t)$

Now, $$d\vec{r_i} = \frac{\partial \vec{r_i}}{\partial q_1} dq_1 + \frac{\partial \vec{r_i}}{\partial q_2} dq_2 + \ldots + \frac{\partial \vec{r_i}}{\partial q_n} dq_n + \frac{\partial \vec{r_i}}{\partial t} dt.$$

$$\boxed{\text{or, } d\vec{r_i} = \sum_{j=1}^{n} \frac{\partial \vec{r_i}}{\partial q_j} dq_j + \frac{\partial \vec{r_i}}{\partial t} dt}$$

The subscript i refers to the particle and has values $i = 1, 2 \ldots N$ and j refers to generalised co-ordinates and has values $1, 2, \ldots n$, dq_j are called the generalised displacement or virtual arbitrary displacement.

Generalised velocity :-

We have $$d\vec{r_i} = \sum_{j=1}^{n} \frac{\partial \vec{r_i}}{\partial q_j} dq_j + \frac{\partial \vec{r_i}}{\partial t} dt \quad (1)$$

Dividing both side by dt in eqn (1), $$\frac{d\vec{r_i}}{dt} = \sum_{j=1}^{n} \frac{\partial \vec{r_i}}{\partial q_j} \frac{dq_j}{dt} + \frac{\partial \vec{r_i}}{\partial t}$$

Now, $\frac{dq_j}{dt} = \dot{q_j}$ is the time derivative of the generalised co-ordinate q_j and represent the generalised velocity.

putting, $\frac{d\vec{r_i}}{dt} = \dot{\vec{r_i}}$, we have,

$$\boxed{\dot{\vec{r_i}} = \sum_{j=1}^{n} \frac{\partial \vec{r_i}}{\partial q_j} \dot{q_j} + \frac{\partial \vec{r_i}}{\partial t}}$$